Problems for Temporary Existence in Tense Logic

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Abstract
A-theorists of time postulate a deep distinction between the present, past and future. Settling on an appropriate logic for such a view is no easy matter. This *Philosophy Compass* article describes one of the most vexing formal problems facing A-theorists. It is commonly thought that A-theories can only be formally expressed in a tense logic: a logic with operators like $P$ (“it was the case that”) and $F$ (“it will be the case that”). And it seems natural to think that we live in a world where objects come to exist and cease to exist as time passes. Indeed, this is typically a key component of the most prominent kind of A-theory, presentism. But the temporary existence assumption cannot be upheld in any tense logic with a standard quantification theory. I will explain the problem and outline the philosophical and logical considerations that generate it. I will then consider two possible solutions to the problem – one that targets our logic of quantification and one that targets our assumptions about change. I survey the costs of each solution.

1. Univocal Existence and Temporary Existence

Metaphysicians should be applauded when they make efforts to be sensible. Here are two particular convictions about existence and change that seem worthy of defense:

**Univocal Existence:** There is just one way that objects exist; and
**Temporary Existence:** Some objects came to exist or will cease to exist.

I suspect before we do much metaphysics we believe both principles, at least in their informal guises. Further, each of the principles enjoys a respectable philosophical pedigree. **Univocal Existence** is the rallying cry of the neo–Quinean movement. Putting the assumption more precisely, neo–Quineans hold that quantificational expressions like “$\exists x$” and “$\forall x$” are how we capture the single, metaphysically important sense of existence. When we wonder what exists in the broadest philosophical sense, we are asking what is in the scope of the broadest universal quantifier. W. v. Quine (1953), David Lewis (1986, Chap 1), Peter Van Inwagen (1998), and Theodore Sider (2009) all defend this more precise univocal existence assumption, and it has come to structure many debates in contemporary metaphysics. To take a common example, suppose you think there is a prime number between two and four. Are you willing to conclude that numbers exist in just the same way as you, me, tables, and electrons exist? If not, according to neo–Quineanism, you must be very careful in how you state your theory that there is a prime between two and four. In particular, you should avoid agreeing to any sentence of the form “$\exists x (\text{Number}(x) \land \text{Prime}(x) \ldots)$” without some qualification.

Some philosophers deny the univocal existence principle, and this is a live debate in metaontology. For example, neo–Meinongians like Terence Parsons (1980) and Graham Priest (2005) deny that a single existential quantifier expression captures a single,
metaphysically important sense of existence. Instead, they propose different kinds of logical apparatus (i.e. multiple fundamental quantifiers or quantifiers and existence predicates) for capturing different metaphysically interesting senses of existence. This apparatus helps them to formulate claims like there are (in one sense) objects which do not exist (in another sense). It is harder to situate the historical Meinong in this debate, but Meinong (1960) gives a theory close to what I am calling “Meinongianism”. Deflationists like Rudolf Carnap (1950), Eli Hirsch (2009) and Thomas Hofweber (2009) think there is no metaphysically privileged sense of existence, so a fortiori there is no privileged logical apparatus for expressing existence. This is another option for resisting the univocal existence assumption. Still I’d venture a guess that these are minority positions in contemporary metaphysics, in part because many of us have trouble making sense of the Meinongian distinctions and in part because – contra the deflationists – many ontological debates seem to be substantive rather than plagued by ambiguity.

The second principle, temporary existence, is traditionally a component of the A-theories of time. A-theorists postulate a deep distinction between the present, past and future. By far the most common variety of A-theory is presentism, the view that only the present is real. There are different ways of making this slogan more precise, but most presentists hold that every non-abstract object is located at, at most, one region of spacetime (the present) and as time passes, objects or regions of spacetime come to exist and cease to exist. Prominent advocates of presentism include Arthur Prior (1967, 1998), John Bigelow (1996), Dean Zimmerman (1998), Peter Ludlow (1999), Thomas Crisp (2003), Ned Markosian (2004), and Trenton Merricks (2007). A smaller, but still notable contingent of A-theorists subscribe to the growing block theory of time. According to the growing block theory, objects or regions of spacetime come to exist but never cease to exist: every non-abstract object is contained in an ever-growing spacetime manifold. C. D. Broad (1923), Robert Adams (1989), Michael Tooley (1997), and Peter Forrest (2006) have all defended such views, and as with presentism, the most common versions of growing block entail that there are temporary existents. There are other varieties of A-theory as well – the shrinking block, the moving spotlight, the dynamic block, to name a few. But there are fewer contemporary defenders of these A-theories.

B-theorists of time are far and away the most prominent critics of the temporary existence principle. They think of change over time in much the way we ordinarily think of spatial variation. According to the B-theory, time is “spread out” in a multi-dimensional manifold. The past/present/future distinction merely distinguishes different parts of the manifold. The present is not metaphysically special: it is merely the part of the manifold when we are. The past is the part of the manifold when dinosaurs are. The future is the part of the manifold when the 55th President of the United States is. These objects scattered throughout the manifold are just as real as objects located in our time, but they are temporally distant. The B-theory strikes many as a radical view, in part because changes like creation, destruction, coming to be and passing away seem to be more profound than mere location change. To the causal observer, it seems dinosaurs are not just hidden in some far corner of spacetime; they have been utterly annihilated. Still the B-theory has proved a powerful contender in the last century, with advocates including Bertrand Russell (1915), W. v. Quine (1950), J. J. C. Smart (1963), David Lewis (1986), Mark Heller (1990), Huw Price (1996), and Theodore Sider (2001).

Temporary, univocal existence seems like a reasonable position to defend. Unfortunately, there are serious challenges facing anyone who wants to be both sensible about time and existence and formally precise. Most A-theorists who endorse both univocal existence and temporary existence use quantified tense logic to express their views. In the
next section, I will explain why supporters of the principles turn to tense logic. Then in Section 3, I will explain a recalcitrant problem for the temporary existence principle: in even the weakest quantified tense logics, temporary existence entails a contradiction. Finally, in Sections 4 and 5, I will survey two responses to the problem. The first response is primarily logical: if we adopt a kind of free logic, then we can avoid the contradiction. The challenge for this solution, as we’ll see, is reconciling the revised tense logic with the univocal existence principle. The second response is primarily metaphysical: if we deny that anything changes with respect to existence, then we can avoid the contradiction. The challenge for this solution, as we’ll see, is finding a plausible substitute for the temporary existence principle.

2. Aliens and Prophylactics

Every so often during Congressional budget season NASA optimistically announces: “There will be a Mars research station in our generation.” We can certainly question the political expediency or scientific viability of the prediction. We might equally wonder about its logical form. Suppose you are a neo-Quinean, presentist space enthusiast. You think the Mars station does not exist. But you trust the government and believe that the station will come to be. What precisely do you believe?

The claim about the Mars station is an example of a claim about a temporal alien, an object that doesn’t exist but once existed or will exist. There seem to be many true claims about temporal aliens. There were dinosaurs. There will be a 55th President of the United States. To capture the logical form of these claims without explicitly quantifying over extinct dinosaurs or yet-to-be-born Presidents, many neo-Quinean A-theorists make use of prophylactic operators. A prophylactic operator is a logical device that blocks the existential commitments of quantifiers in its scope. Prophylactic operators are a familiar tool in philosophical logic. The truth-functional negation operator is the ultimate prophylactic – a quantified sentence in the scope of negation commits us to non-existence. But there are also common non-truth-functional prophylactics, like mere possibility and mere fiction operators. For example, few of us think that “In the Battlestar Galactica fiction, there is a Cylon” entails that a Cylon exists. Rather, it seems natural to suppose that Cylon existence is only true according to the Battlestar fiction. We can try to codify this distinction in logic. Let $B$ abbreviate the “in the Battlestar fiction” operator. The Cylon claim is ontologically innocent because when we formulate it more precisely – $B \exists x Cx$ – the existential quantifier is qualified by the prophylactic $B$. Neo-Quinean Battlestar fans rest easy so long as $B \exists x Cx$ does not entail any unprotected existential claim like $\exists x Cx$ (something is a Cylon) or $\exists x BCx$ (something is such that, according to Battlestar, it is a Cylon).

To account for temporal aliens, many A-theorists extend the prophylactic assumption to tense operators like $P$ (short for “it was the case that”) and $F$ (“it will be the case that”), and then treat the past and future much like we treat merely possible worlds or fictions. For example, many Neo-Quinean A-theorists formulate the Mars station claim as $F \exists x Sx$ (it will be the case that there is a Mars station). The $F$ operator allows us to safely make predictions about space stations so long as $F \exists x Sx$ doesn’t entail any unprotected quantified claims like $\exists x Sx$ (something is a Mars station) or $\exists x F Sx$ (something is such that it will become a Mars station). A-theorists put a lot of stock in the prophylactic powers of certain tense operators. Hence Zimmerman:
The presentist must, I think, be a serious tenser. At the very least, tenseless statements that require ontological commitment to past and future things must be treated as equivalent to tensed truths that do not. (1998: 211).

And much of the foundational work on the A-theories presupposes a tight formal analogy between tense and modality. As should be clear, the prophylactic operator strategy forces adherents to develop views on intensional logics, because anyone who uses the strategy to express their views must have an account of the entailment relations between prophylactic and non-prophylactic sentences. The translation scheme would be useless if allegedly prophylactic sentences always entailed existentially loaded ones. I will call a given logic adequate for a metaphysical theory just in case that theory can be consistently expressed in the logic. Most actualists and Neo-Quinean Battlestar fans assume there is a modal logic adequate for their metaphysical theories. Neo-Quinean A-theorists assume there is an adequate tense logic for capturing truths about dinosaurs, space outposts, and other temporal aliens.

Unfortunately, it is not at all clear that prophylactics work. The modal logic debate has long been plagued by problems involving the Barcan schemas. Here is an overview of the most discussed version of the Barcan problem. Many metaphysicians find it reasonable to suppose that if something is possible, it is necessarily possible. Put another way, every possible world is possible relative to every possible world. The S5 System of modal logic formalizes this assumption. But if we extend S5 with the most straightforward logic for quantifiers, then the following schema is counted a logical truth: $\Diamond \exists x \rightarrow \exists x \Diamond z$. This result was first shown by Ruth Barcan-Marcus in the 1940s, and it subsequently set the agenda for decades of work on modal metaphysics. Given the Barcan schema, allegedly safe sentences like “It is possible for a Cylon to exist” ($\Diamond \exists x Cx$) entail existentially loaded ones like “Something is a possible Cylon” ($\exists x \Diamond Cx$). The first claim is reasonable enough, but only a paranoid would believe the second (More accurately, only a paranoid or someone with a very liberal conception of possible properties would believe it.).

There is a temptation to think that the Barcan schema is an isolated problem for modal metaphysics, because A-theorists need not assume the correct tense logic is an extension of S5. Indeed, for some a philosophical motivation for the A-theory is the conviction that the future is open, and so some propositions about the future are not accessible from others. Imagine time as a tree, with possible futures branching off of the present. Distinct future branches are not in the past or future with respect to other branches. Rather they represent independent, mutually exclusive futures. But without assuming symmetry (and the corresponding tense logic), the regular Barcan schema doesn’t hold for $P$ or $F$. This may seem like a reason to be optimistic about the prospects for a metaphysically adequate tense logic, at least more optimistic than we are about the prospects for a metaphysically adequate modal logic. After all, the most famous problem for modal logic will not obviously carry over to tense logic. But as we will see in the next section, complacency is unwarranted.

3. Simple Tense Logic and Difficulties for Temporary Existence

We need not assume that the correct tense logic is a variant of S5, but what is an adequate tense logic? Suppose we start from scratch, trying to build a tense logic for the A-theories by adding axioms and rules for tense operators to standard predicate logic. This is an initially promising idea: predicate logic has proven its worth time and again in metaphysics. But we quickly find that in any tense logic based on a standard quantification
theory, the temporary existence principle is false. To see why, let’s look in some detail at the simplest such tense logic, Quantified Tense Logic K (QTLK). I’ll outline the apparatus of the logic to give you a sense of how it works and highlight the particular components that give rise to the problem.

For our purposes, a tense logic is any formal system that includes the four Priorian tense operators, two of which we have already met, the prophylactics:

\[ P: \text{“it was the case that”}; \]  
\[ F: \text{“it will be the case that”}. \]

We add to this list the two “always” operators, which formally correspond to the necessity operator in modal logic:

\[ H: \text{“it has always been the case that”}; \]  
\[ G: \text{“it will always be the case that”}. \]

QTLK results from adding axioms, definitions, and rules for these tense operators to a standard predicate logic with identity. For the predicate logic, we’ll assume the usual axioms for propositional logic (abbreviated PL) and add seven schemas, definitions and rules for reasoning with quantifiers and identity. This will give us a logic of existence.

*Standard Predicate Logic (with Identity):*

Where \( \alpha \) and \( \beta \) are any wffs and \( x \) and \( y \) are any variables...

[Def\( \exists x \)] \( \exists x \alpha \equiv \neg \forall x \neg \alpha. \)

[\forall 1] \( \alpha[y/x] \) is \( \alpha \) with free \( y \) replacing every free \( x \), then \( \forall x \alpha \rightarrow \alpha[y/x] \) is an axiom.

[\forall 2] \( \forall x (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \forall x \beta) \) is an axiom, as long as \( x \) does not occur free in \( \alpha \).

[=1] \( x = x \) is an axiom.

[=2] If \( \alpha \) and \( \beta \) differ only in that \( \alpha \) has \( x \) free wherever \( \beta \) has \( y \) free, \( x = y \rightarrow (\alpha \rightarrow \beta) \) is an axiom.

[Modus Ponens] If \( \alpha \) and \( \alpha \rightarrow \beta \) are theorems, then \( \beta \) is a theorem.

[Universal Generalization] If \( \alpha \) is a theorem, then \( \forall x \alpha \) is a theorem.

We define a theorem as any axiom or any line in a proof that is derived from a rule of inference and other axioms or theorems. Of these seven, \([\forall 1]\) plays the most important role in the problems to come.

To get QTLK, we then include seven new schemas, definitions, and rules for handling the tense operators:

[Def\( P \)] \( P \alpha \equiv \neg H \rightarrow \alpha. \)

[Def\( F \)] \( F \alpha \equiv \neg G \rightarrow \alpha. \)

[K1] \( H (\alpha \rightarrow \beta) \rightarrow (H \alpha \rightarrow H \beta) \) is an axiom.

[K2] \( G (\alpha \rightarrow \beta) \rightarrow (G \alpha \rightarrow G \beta) \) is an axiom.

[K3] \( \alpha \rightarrow HF \alpha \) is an axiom.

[K4] \( \alpha \rightarrow G \alpha \) is an axiom.

[Eternalization] If \( \alpha \) is a theorem, then \( H \alpha \) and \( G \alpha \) are theorems.
Eternalization is the tense logic equivalent of the Necessitation rule in modal logic. We assume all axioms and theorems in the system are always true. (Remember this only applies to lines of the proof that are axioms or derived from only axioms and rules. We cannot use Eternalization if we are reasoning from other assumptions.)

How do we determine which wffs are logical validities in the simplest tense logic? The corresponding semantics for QTLK is formally elegant, if metaphysically tendentious. A model \( M \) is a quadruple \((T, R, D, I)\). \( T \) is a non-empty set; informally, \( T \) is a domain of times. \( R \) is a binary relation on members of \( T \); informally, \( R \) tells us which times are past or future relative to others. \( D \) is another non-empty set; informally, \( D \) is the domain of existents. And \( I \) is an interpretation function that maps every \( n \) place predicate in every wff to a set of \( n + 1 \) tuples; informally, \( I \) gives the extension of every predicate relative to a time. In determining the mapping, the first \( n \) objects are drawn from \( D \), and the last is drawn from \( T \). This model structure is metaphysically problematic because it is unclear how we should understand the set of times if we think non-present times do not exist. Most likely the times will need to be treated as some kind of abstracta that represent the past and future. Deeper issues lurk behind this move, but having flagged the worry, let me now set it aside and simply recommend further reading on the topic.

With this model structure in hand, we then use a valuation function, \( V \), to determine one of two truth-values (0 or 1) for every wff relative to a model. Our valuation function obeys the following constraints. Where \( \phi \) is any predicate, \( \forall x \) and \( \forall y \) are any wffs, and \( \mu \) is an assignment function that maps every variable to a member of \( D \):

\[
[V\phi] \quad V_{M,\mu}(\phi(x_1,\ldots,x_n), t) = 1 \text{ if } \langle \mu(x_1), \ldots, \mu(x_n), t \rangle \in I(\phi) \text{ and 0 otherwise.}
\]

(Informally: consider an atomic wff, a predicate with \( n \) argument places filled by \( n \) variables. That wff is true relative to a model and a variable assignment at a time \( t \) just in case the \( n \)-tuple of objects assigned to the variables at \( t \) is in the extension of the relevant predicate at \( t \). Otherwise it is false.)

\[
[V=] \quad V_{M,\mu}(x = y, t) = 1 \text{ if } \mu(x) = \mu(y) \text{ and 0 otherwise.}
\]

\[
[V\neg] \quad V_{M,\mu}(\neg z, t) = 1 \text{ if } V_{M,\mu}(z, t) = 0 \text{ and 0 otherwise.}
\]

\[
[V\rightarrow] \quad V_{M,\mu}(z \rightarrow \beta, t) = 1 \text{ if } V_{M,\mu}(z, t) = 0 \text{ or } V_{M,\mu}(\beta, t) = 1 \text{ and 0 otherwise.}
\]

\[
[V\forall] \quad V_{M,\mu}(\forall x \phi, t) = 1 \text{ if } V_{M,\mu}(\phi, t) = 1 \text{ for all assignments } \mu^* \text{ and for all objects } o, \text{ such that } o \in D \text{ and 0 otherwise. A } \mu^* \text{ assignment is identical to } \mu \text{ except in assigning } o \text{ to } x. \text{ Informally: for a universally quantified wff of the form } \forall x \phi \text{ to be true at a time, } x \text{ must be true at that time for every object in the domain that could be assigned to } x.]
\]

\[
[V\exists] \quad V_{M,\mu}(\exists x \phi, t) = 1 \text{ if } V_{M,\mu}(\phi, t) = 1 \text{ for some assignment, } \mu^* \text{ and some } o \in D \text{ and 0 otherwise.}
\]

\[
[VP] \quad V_{M,\mu}(Pz, t) = 1 \text{ if } V_{M,\mu}(z, t') = 1 \text{ for some } t' \text{ such that } R(t', t) \text{ and 0 otherwise.}
\]

\[
[VF] \quad V_{M,\mu}(Fz, t) = 1 \text{ if } V_{M,\mu}(z, t') = 1 \text{ for some } t' \text{ such that } R(t, t') \text{ and 0 otherwise.}
\]

\[
[VH] \quad V_{M,\mu}(Hz, t) = 1 \text{ if } V_{M,\mu}(z, t') = 1 \text{ for all } t' \text{ such that } R(t, t') \text{ and 0 otherwise.}
\]

\[
[VG] \quad V_{M,\mu}(Gz, t) = 1 \text{ if } V_{M,\mu}(z, t') = 1 \text{ for all } t' \text{ such that } R(t, t') \text{ and 0 otherwise.}
\]

A wff \( \phi \) is true in a model \( M \) if and only if \( V_{M,\mu}(\phi) = 1 \) on each variable assignment for \( M \). A wff is valid if and only if it true on all models at all times. As with the proof theory, the most controversial part of this semantics is its treatment of the quantifiers in clauses \([V\forall]\) and \([V\exists]\).

So much for QTLK’s formal apparatus. Note that this is as simple a quantified tense logic as you can get. It has relatively few axioms, and so far we have put no restrictions...
on the R relation in the semantics. Of course, most defenders of temporary existence will want to further flesh out the logic to reflect their other metaphysical commitments. Advocates of the growing block theory might dispense with the axioms and clauses for P and H. Given that they freely assert the existence of merely past objects and times, they have no direct need for past prophylactic operators. And anyone who believes in the open future or believes that the past is not fixed will somehow need to amend [K3] and [K4] and the semantic theory that validates them. [K3] rules out the most obvious formulations of the open future intuition, and [K4] rules out changing the past. But set aside these worries about implementation, since we have a more sweeping issue to confront.

We are now in a position to show why QTLK wreaks havoc on our two sensible assumptions about time and existence. In QTLK, we can prove the converse tense Barcan schemas:

[CB-Past] \( H \forall x \alpha \rightarrow \forall x H \alpha \); and
[CB-Future] \( G \forall x \alpha \rightarrow \forall x G \alpha \).

The proofs of these schemas are pretty straightforward. I'll sketch one, making use of a derived rule in QTLK. Here's [CB-Past]:

1. \( \forall \alpha \forall x \alpha \rightarrow \alpha \). [\( \forall 1 \)]
2. \( \vdash H(\forall x \alpha \rightarrow \alpha ) \rightarrow (H \forall x \alpha \rightarrow H \alpha ) \). [A version of a rule we derive from [K1]:
   \( \vdash H(\alpha \rightarrow \beta ) \rightarrow (H \alpha \rightarrow H \beta ) \)]
3. \( H(\forall x \alpha \rightarrow \alpha ) \). [Eternalization], 1
4. \( H \forall x \alpha \rightarrow H \alpha \). [Modus Ponens], 2, 3
5. \( \forall \alpha (H \forall x \alpha \rightarrow H \alpha ) \). [Universal Generalization], 4
6. \( \forall x (H \forall x \alpha \rightarrow H \alpha ) \rightarrow (H \forall x \alpha \rightarrow \forall x H \alpha ) \). [\( \forall 2 \)]
7. \( H \forall x \alpha \rightarrow \forall x H \alpha \). [Modus Ponens], 5, 6

We can perform exactly the same style of proof for [CB-Future]. And both schemas are demonstrably valid in the semantics of QTLK.7

Now suppose that at least one object hasn’t always existed. How do we express this in the language of QTLK? Faithful neo-Quineans will initially express it using quantifiers and identity. One of several equivalent ways to do this is as:

1. \( \neg \forall x H \exists y(x = y) \).

In English: it is not the case that everything has always been something. But by [CB-Past], the following is a theorem:

2. \( H \forall x \exists y(x = y) \rightarrow \forall x H \exists y(x = y) \).

From (1), (2) and modus tollens we can conclude:

3. \( \neg H \forall x \exists y(x = y) \).
In English: it isn’t the case that it has always been that everything is something. But this is madness … surely always everything is something! This follows from univocal existence and the interdefinability of the quantifiers. And indeed, $H \forall x \exists y (x = y)$ is yet another theorem of QTLK. We get it by applying Eternalization to the proof that everything exists in regular predicate logic with identity. I’ve sketched that proof in an endnote. Parallel reasoning applies for [CB-Future] and the claim that some object will cease to exist. In the simplest tense logic with a standard quantification theory, temporary existence claims lead straight to a contradiction.

This problem is not often addressed in contemporary work on time, but (as noted in Section 2) related issues have played a prominent role in debates over modal metaphysics and can be found in some of the early work on the A-theory. For example, Prior discusses issues with the Barcan schemas in Chapter 8 of *Past, Present, and Future*, and more general problems accounting for temporary existence partly motivate his awkward System Q tense logic. Contemporary A-theorists have largely adopted the language of tense logic, without resolving this crucial background problem. So can contemporary A-theorists succeed where Prior left off? There are two main lines of response to the problem, each suggested by kinds of response favored in the parallel debate over necessary and contingent existence and the regular Barcan schemas. The first line of response, inspired by Saul Kripke, tries to save our metaphysical convictions by amending the quantification theory. I’ll discuss this line of response and some challenges to it in the next section. The second line of response, inspired by Timothy Williamson, tries to avoid the problem by proposing an alternative to the temporary existence principle. We’ll consider this proposal in Section 5.

4. Option 1: Adopt a Different Logic for Quantifiers?

Faced with the strange entailments of the converse Barcan schemas, we may be first inclined to blame the logic used to derive them. For example, here is how John Burgess responds to those like Williamson who would consider rejecting temporary existence once they realize that it is false in QTLK:

What is wrong with this reply is not that the metaphysical doctrine… is silly (silly though it is). What is wrong is making logic depend on metaphysical doctrine, and that would be wrong even if the doctrine were more sensible. Temporal logic ought rather to be able to offer physicists and metaphysicists alike all the options, so that adopting any particular doctrine, silly or sensible, would show up in the validity of some additional law, not part of the minimal logic. (2009: 36)

So can we rewire tense logic to accommodate the temporary existence principle? Here we benefit from the considerable work done on the Barcan schemas in modal logic. The most popular strategy for addressing the Barcan problems in modal logic targets the quantification theory. In particular, many blame the problematic schemas on axiom $[\forall 1]$, the axiom that allows us to strip quantifiers off of certain wffs. Recall the axiom:

$[\forall 1]$ For any variables $x$ and $y$, if $\mathcal{A}[y/x]$ is $\mathcal{A}$ with free $y$ replacing every free $x$, then $\forall x \mathcal{A} \rightarrow \mathcal{A}[y/x]$ is an axiom.
The rough idea behind $[\forall 1]$ is that at least one particular truth answers to every universally quantified claim. One well-known and controversial consequence of $[\forall 1]$ is that $\exists x(x = x)$ is a theorem of standard predicate logic; it is impossible to have an empty domain (see endnote 8 for proof). For this reason, among others, free logicians contend that $[\forall 1]$ is intuitively invalid. Why, they wonder, should it be decided by logic alone that at least one object exists? To avoid this result, free logicians propose an alternative elimination schema:

$$[\forall 1 \text{ Free}] \text{ For any variables } x \text{ and } y, \text{ if } z[y/x] \text{ is } z \text{ with free } y \text{ replacing every free } x, \text{ then } \forall y(\forall x z \rightarrow z[y/x]) \text{ is an axiom.}$$

The new schema is distinctive in that it only allows closed formulae as theorems. And without open formulae, the proof of $\exists x(x = x)$ fails.

Kripke was the first to realize that the free logic quantification theory could also help block the problematic Barcan schemas. For somewhat technical, tangential reasons, a different axiom is used to develop free modal logics. It involves an extra universal quantifier:

$$[\forall 1 \text{ Free}'] \text{ For any variables } x,y \text{ and } z, \text{ if } z[y/x] \text{ is } z \text{ with free } y \text{ replacing every free } x, \text{ then } \forall y\forall z(\forall x z \rightarrow z[y/x]) \text{ is an axiom.}$$

Call a tense logic structurally similar to QTLK but with $[\forall 1 \text{ Free}']$ instead of $[\forall 1]$ the simplest free tense logic. Free tense logicians will not need to amend any other parts of the proof theory, but they should restrict the application $[\forall 2]$ and rules like Necessitation and Eternalization to closed formulae. What happens if we shift to the simple free tense logic? As we saw in Section 3, the proofs of the converse Barcan schemas depend on $[\forall 1]$. So these proofs are effectively blocked. We will also be unable to prove more innocuous theorems like $\forall x \exists y (x = y)$ (everything is something), but perhaps this is a result that neo-Quineans can bear. Is some free tense logic the best option for A-theorists? The answer to this question depends on how seriously you take the formal semantics that accompanies free logics.

Suppose you think it is not enough that the converse Barcan schemas are not provable: a proper logic for time ought to show that they are invalid. When we move to free logic’s more inclusive quantification theory, we also have to adopt a new semantics, in this case a variable domain semantics. Rather than filling out models with objects from a single domain, a variable domain semantics postulates a “superdomain” and generates many domains of different sizes by drawing from the members of the superdomain. Informally, our new semantics is going to allow each time in our model to have its own domain of objects, drawn from a superdomain. More formally, a model for free tense logic is a quintuple $(T, R, D, Q, I)$. $T$ and $D$ are non-empty sets, just as before, but in the variable domain semantics $D$ is not to be identified with the set of objects that exist without qualification. (More on this in a moment.) $D$ is the superdomain. $R$ is still a relation on members of $T$. $Q$ is a function that assigns subsets of $D$ to every time in $T$, resulting in a domain for that time, $D_t$. And $I$ is an interpretation that maps each n place predicate in every wff to a set of $n + 1$ tuples. The clauses for determining truth-values are the same as before, except the clauses for quantifiers, which change to:
Unlike before, now our interpretation will only draw from the domains generated by the Q function when it is filling in the first \( n \) places.

Within the variable domain semantics we can show that the converse Barcan schemas are invalid. Consider one of the metaphysically troublesome versions of \([\text{CB-Future}]: \forall \exists y (x = y) \rightarrow \forall x G \exists y (x = y)\). This version states that if it is always going to be the case that everything is something, then everything is always going to exist. We want this conditional to be false on at least one model. Let \( M \) be a model with the following features. The superdomain, \( \mathcal{D} \), has just two members: \( a \) and \( b \). The set of times has two members: \( t_1 \) and \( t_2 \). \( Q \) generates domains as follows: the domain of \( t_1 \) is \( \{a, b\} \) and the domain of \( t_2 \) is just \( \{a\} \). \( R \) imposes a symmetric relation on \( t_1 \) and \( t_2 \): \( \{(t_1, t_2), (t_2, t_1)\} \). At \( t_1 \) it is true that it is always going to be that everything is something. But at \( t_1 \) it is not true everything is always going to exist, because \( t_2 \) is accessible from \( t_1 \) and \( b \) is not in the domain of \( t_2 \). So the problematic version of the Barcan schema is false on this model. I find diagrams are often helpful in seeing how the countermodels work. Here is a diagram for this countermodel, where the arrows represent that each time is \( R \) related to the other (Figure 1). Since there is at least one model with one time and assignment such that this instance of \([\text{CB-Future}]\) is false, \([\text{CB-Future}]\) is invalid. And we can construct a parallel countermodel for \([\text{CB-Past}]\).

The variable domain semantics delivers the desired formal results: the converse Barcan schemas are demonstrably not logical truths. But it is far less clear whether the proposal meets the demands for metaphysical adequacy. Have we done justice to our original assumptions about time and existence? Remember that the univocal existence principle holds that there is just one metaphysically important sense of “exists.” Yet on the variable domain semantics, quantifiers are only evaluated with respect to the distinct domains generated by the Q function. In the semantics, there is no such thing as existence or non-existence full stop. There is merely existence relative to one of the many domains \( D_1, D_2, \ldots \) etc. Something – for example \( b \) in our countermodel above – can “exist” according to \( D_1 \) and not “exist” according to \( D_2 \). So in the variable domain framework, it makes no sense to ask whether an object like \( b \) exists in the single, broadest sense of existence. Whatever sense of existence the semantics is formalizing, it cannot be the sense of interest to neo-Quinean metaphysicians. This is the primary objection to free tense logic: it does not model the sense of existence metaphysicians are interested in.

\[
\begin{align*}
[\forall \exists] & V_{M,a}(\forall x a, t) = 1 \text{ if } V_{M,a}^*(x, t) = 1 \text{ for all assignments } \mu^* \text{ and all } o \in \mathcal{D}_t \text{ and } 0 \text{ otherwise. [Now our assignment function is only drawing from } \mathcal{D}_c.]
\end{align*}
\]

Fig. 1. A diagram of a countermodel to the tensed converse Barcan schema.
In responding to this objection, the free tense logic A-theorist has two options, each with accompanying challenges. First the A-theorist might adopt the variable domain semantics but deny that the univocal existence assumption applies to it. In “Two Axes of Actualism”, Karen Bennett suggests a strategy like this for understanding the quantificational theory for modal logic (308–9). Perhaps when we use quantifiers to express an explicit metaphysical theory, we commit to them picking out the single, important sense of existence. But when we are evaluating quantifiers in a formal model, we no longer hold them to this standard. Defenders of this solution to the problem must then somehow justify the special exception they give for quantifiers in the formal semantics. What is the point of the formal quantification theory if it is not describing the behavior of the single, metaphysically important senses of “exists”?

The A-theorist might also simply reject the variable semantics and shrug off the demand to demonstrate that the converse Barcan schemas are invalid. Operator primitivists hold that we cannot give accurate, purely compositional, reductive semantic clauses for intensional operators. Suppose you were to combine operator primitivism with the proof theory in free tense logic. The operator primivists do not need to qualify the univocal existence principle, because they do not assume that we evaluate quantifiers with respect to different domains. But they dodge the problem only by failing to provide any semantics to invalidate the converse Barcan schemas. How bad is this result? Your answer to this question depends on how important you think the formal semantics is for determining the metaphysical adequacy of a logic. Has the supporter of temporary existence found an adequate logic just so long as the converse Barcan schemas cannot be derived in her proof theory? Or does she have a burden to provide a theory on which they are demonstrably not logical truths? Operator primitivists can meet the first challenge, but they cannot meet the second. This is the main cost of adopting primitivism.

Note that B-theorists do not face these kinds of tradeoffs with their logic and semantics. If you think past and future times and objects exist, you can freely quantify over them. B-theorists can express their views in standard predicate logic with standard semantic clauses for quantifiers. Unlike the free logic A-theorists we are considering, they have no special difficulty demonstrating which truths about time are logically valid or invalid.

Recall Burgess’s quote from the start of the section. Burgess assumes that there is a metaphysically neutral minimal logic. As we’ve seen, when we delve deeper into our options for a quantification theory, this assumption is far from obvious. As we make our models more inclusive, we lose intuitive theorems like \( \forall x \exists y (x = y) \), and we are pressured to either cheat on our application of the univocal existence principle or accept that we cannot we invalidate the troublesome schemas. Controversial choices come with every formal option.

5. Option 2: Deny Temporary Existence?

Unwilling to pay the costs associated with free tense logic, some philosophers have urged that we keep QTLK and univocal existence but rethink the notion of temporary existence that leads to the contradiction. Timothy Williamson is perhaps the most prominent advocate of a metaphysics-based response to the Barcan schemas. In a series of recent papers, he argues that necessary, eternal existence is not as anathema to sensible metaphysics as one might initially suppose (see Williamson (1998, 2000a,b, 2002, 2010)). One way Williamson motivates his view is by reformulating some of our ordinary descriptions of change. For example, in “Bare Possibilia” he asks us to consider a case involving a river that has dried up, the river Inn:
Always everything is something. Therefore by (the converse Barcan schema), everything is always something... What kind of thing has the Inn become, if it is no longer a river? Given that abstractness is not a temporary property, it has not become an abstract object. The best and most natural answer is just that the Inn was once a river; it is a past river. Its characteristic properties concern its past; whether it continues to leave traces in the present is inessential to its nature. (265–6).

According to Williamson, when objects are created or destroyed, they do not change with respect to existence. Rather they undergo a form of radical property change, gaining or losing all of their intrinsic present properties. The Inn changes from being a merely future river, to being a river, to being a merely past river. But it never ceases to be full stop. Williamson is still an A-theorist, since he draws a sharp distinction between the present and other times. But he denies that formulae like $\neg\forall x H \exists y (x = y)$ accurately capture what we believe when we believe that an object “came to be”. He does not distinguish the present ontologically.

Williamson’s approach sidesteps the formal problem. Without assumptions like $\neg\forall x H \exists y (x = y)$, QTLK does not generate a contradiction. And a permanent existence A-theory might have some independent metaphysical advantages by accommodating some of the objections that lead others to eternalist B-theories. For example, it might help us make sense of why names for temporal aliens like “Elvis” or “Julius Caesar” seem to still refer. Nevertheless, many balk at the suggestion that creation and destruction are merely property changes. Those who would adopt this substitute for temporary existence face two kinds of cold, incredulous stares. First, Williamson’s view requires a somewhat radical anti-essentialism. An essential property (on one common construal) is a property such that an object could not lack it. According to Williamson’s view, you will continue exist even after you have lost the property is rational. Dinosaurs continue to exist even though they have neither DNA nor any spatiotemporal location. Second and relatedly, the view seems unjustifiably ontologically inflationary. If Williamson is right, we must conclude Caesar and the whole host of former Roman emperors exist, even though none of these objects causally interact with any of us now. Stranger still, something that exists right now will become the 55th President of the United States, even though that thing is not (now) a natural-born citizen nor even a human being. So why take univocal existence seriously if we are forced to admit so many mysterious objects into our ontology? Those who would adopt Williamson’s solution to the problem must rethink common metaphysical assumptions about essence and persistence.

6. Conclusion

Univocal existence and temporary existence seem to be highly plausible, defensible metaphysical principles. But upholding both in a consistent tense logic is quite difficult, and as we have seen, A-theorists face some awkward tradeoffs. Without a theory of tense operators, we cannot capture truths about merely past and future objects. In the simplest quantified tense logic, the temporary existence principle is false. In free tense logic, we struggle to make sense of the univocal existence principle. Considerations from logic show that at least one of these two principles is not as straightforward as it seems.16

Short Biography

Meghan Sullivan is an Assistant Professor in the University of Notre Dame’s Philosophy Department. Her research focuses primarily on the metaphysics and logic of time. Her
forthcoming paper “The Minimal A-Theory” argues for a kind of eternalist A-theory. She also has forthcoming work on the philosophy of religion, in particular on how divine name words might co-refer or cease to refer over time. Before coming to Notre Dame, Sullivan earned a PhD at Rutgers University, and a BPhil from Oxford, where she studied as a Rhodes Scholar.

Notes

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1 For example McCall (1994) defends a shrinking block A-theory. Van Inwagen (2010) describes a model of a growing and shrinking block that can be used to capture intuitions about causally efficacious time travel, though he does not endorse the view. Williamson develops a modern variant of the moving spotlight or “eternalist” A-theory. His view will be considered in some detail in Section 5.

2 There are some fictional realists like Thomasson (1999) who admit the existence of fictional entities like Cylons. Thomasson treats fictional objects as abstract artefacts.

3 For example, see Prior’s Time and Modality and Past, Present, and Future where he develops tense logic and argues for it by analogy with the prevailing modal logics of the time.

4 Zimmerman is one example of an open future presentist. For some discussion of the motivations for adopting an open future view, see his “Yet Another Anti-Molinist Argument.” Rea (2006) argues that presentists must adopt an open-future theory or deny libertarian freedom.

5 Zalta (1987) offers one of the most explicit theories of times as abstracta in tense logic. For broader accounts of times as abstract objects, see Bigelow (1996), Zimmerman (1997), and Crisp (2007). For a sympathetic overview of uses for abstracta in the modal debate see Bennett (2005).

6 For our purposes, we’ll assume the only terms are variables. Adding constants makes the clause for atomics sometimes more complicated.

7 Suppose for reductio there is a model M, a time t, and an assignment μ such that $V_{M, μ}(H \forall x x \rightarrow \forall x H x, t) = 0$. By $[V \rightarrow ]$, $V_{M, μ}(H \forall x x, t) = 1$ and $V_{M, μ}(\forall x H x, t) = 0$. Applying $[V H]$ to the first conjunct, $V_{M, μ}(\forall x x, t) = 1$ for all t such that $R(t', t)$. And applying $[V V]$ to this, we can infer $V_{M, μ}(x, t) = 1$ for all t such that $R(t', t)$ and all assignments $μ$. Now consider the second conjunct. Applying $[V V]$, $V_{M, μ}(H x, t) = 0$ for at least one assignment $μ$. And applying $[V H]$, $V_{M, μ}(x, t) = 0$ for at least one time such that $R(t', t)$ and one assignment $μ$. Contradiction. So there is no such model.

8 A sketch of a proof of $\forall x \exists y(x = y)$, using a familiar derived rule of propositional logic, Modus Tollens:

1. $y = y$. $\Box$$\forall x \exists y(x = y)$, using a familiar derived rule of propositional logic, Modus Tollens:
2. $\forall y(x = y) \rightarrow \neg(y = y)$. $[\forall V$ with free y replacing x$]$
3. $\neg\forall y(x = y)$. $[\neg\forall V, \forall y(x = y)]$, using a familiar derived rule of propositional logic, Modus Tollens, 1, 2
4. $\exists y(x = y)$. $[\neg\forall V, \forall y(x = y)]$
5. $\exists y(x = y)$. $[\exists V, \exists y(x = y)]$

9 A majority of those working in modal metaphysics seem to be variable domainers in one respect or another. Some representative examples include Salmon (1987), Menzel (1990), and Bennett (2005).

10 For an overview of some philosophical motivations for free logic, see Lambert (2001, 2003) and Chapter 8 of Bostock (1997).


12 See Fine (1983) for a proof of the problem $[\forall V$ Free$]$. If we drop the identity schemas from free intensional logics (as some philosophers wish to), we can no longer prove they are complete with respect to the variable domain semantics (to be introduced shortly). But in a logic with $[\forall V$ Free$]$ we can. See Hughes and Cresswell (1983: 305). Not much hinges on this result for our purposes.

13 There is a related objection to free modal logic that is more often discussed. We find it in Plantinga(1976), Lin- sky and Zalta (1994), Williamson (1998), and Peacocke (2002). Priest also contends that variable-domain modal logic violates the spirit of actualism, but he takes this as a reason to be more open to an existence predicate; see Priest (2005: 14).


15 For discussion of ways to manage such worries, see Sullivan (forthcoming).

16 Thanks to Ross Cameron, Andy Egan, Jenn Wang, Dean Zimmermann, an anonymous reviewer and especially Ted Sider for advice on parts of this paper.
Works Cited


